The probability amplitude equation of quantum entanglement: the Australian connection

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The origin of the ubiquitous quantum entanglement probability amplitude equation, for the polarisation of photons traveling in opposite directions, is described.

In a recent paper [1], originally written to explain the origin of experimental configurations to determine the quantum entanglement of photons traveling in opposite directions, the issue of the corresponding probability amplitude surfaced. As it turns out, Australian physicists J C Ward and R H Dalitz played key roles in this field, roles that are largely unrecognised, or unknown, by the physics community. Here, a succinct description of these crucial contributions is provided.

The initial discussion on the use of quantum theory to describe the polarisation correlation of quanta propagating in opposite directions was given by Wheeler in 1946 [2]: "According to the pair theory, if one of these photons is polarized in one plane, then the photon that goes off in the opposite direction with equal momentum is linearly polarized in the perpendicular plane." This is the essence of entanglement. The pair theory that Wheeler refers to is the Dirac theory of electron-positron pairs [3].

Ward in 1949 [4] mentions Wheeler's contribution and then continues to explain that Wheeler did attempt to calculate this effect *but* "through the neglect of interference terms he derived an incorrect, and in fact, far too small value for the angular correlations of the scattered quanta" [4]. Ward's thesis [4] includes the physics used to derive the quantum formula for correlated polarisations published by Pryce and Ward in 1947 [5] which was also independently published by Snyder *et al.* [6] in 1948.

Figure 1. J. C. Ward (1924-2000), one of the contributors to the Standard Model and the creator of the Ward identities. His momentous physics career began in 1947 with a brief paper in Nature that correctly predicted the polarisation correlation of entangled photons propagating in opposite directions. English born, and Oxford educated, John Ward first traveled to Sydney in 1947, returned a few years later to Adelaide, for brief period, and finally accepted a professorship at Macquarie in 1964. Between Adelaide and Macquarie, he had several positions in the United States and one in New Zealand. In later years, he traveled extensively with an Australian passport.

Ward's approach begins by listing the polarisation alternatives related to x and y polarisation axes related to two counter propagating photons: $|x,x\rangle, |x,y\rangle, |y,x\rangle, |y,y\rangle$ [4]. Here, the first coordinate refers to photon 1 and the second coordinate to photon 2. Eventually, Ward [4] arrives at the probability amplitude for entangled polarisations $|\psi\rangle = (|x,y\rangle - |y,x\rangle)$. Using the identity given by Dirac $(|x\rangle|y\rangle = |x,y\rangle)$ [7] , once normalised, this expression can be restated as

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle_1|y\rangle_2 - |y\rangle_1|x\rangle_2\right) \qquad (1)$$

This is the probability amplitude used, to derive the polarisation correlation of two propagating quanta in opposite directions, by Pryce and Ward [5] and Snyder *et al.* [6].

Moreover, this is also the famous probability amplitude for entangled polarisations as utilised in optical EPR-Bell type experiments [8].

In his memoirs Ward [9] also mentions that, following the Price-Ward disclosure in *Nature* [5], a young Australian physicist, and Ward's friend, R. H. Dalitz independently derived these results and presented them at a seminar in Cambridge, late in 1947, thus making "quite a name for himself" [9] (Note: for the sake of completeness, Dick Dalitz is known in particle physics for the *Dalitz plot, Dalitz poles*, and the *Dalitz pair*).

In 1950, Wu and Shaknov [10] reported measurements on polarisation correlation of counter propagating photons which agreed, within ~ 2%, with the theoretical value applicable to the geometry of their experimental configuration. In their paper, Wu and Shaknov [10] explicitly state: "As early as 1946 J. A. Wheeler proposed an experiment to verify a prediction of pair theory, that the two quanta emitted in the annihilation of a positron-electron pair, with zero angular momentum, are polarized at right angles to each other... The detailed theoretical investigations were reported by Pryce and Ward and Snyder *et al.*" Further, in 1975 Wu and colleagues wrote: "the explicit expression for the probability of detecting a pair of scattered photons in this geometry... as in eq. (1) was first worked out by Pryce and Ward" [11].

Bohm and Aharanov [12] in 1957 state that the Wu and Shaknov [10] experiment is considered as an example of an EPR type experiment. When this interpretation was brought into question [13], Bohm and Aharanov rejected such criticism [14]: "In a previous paper [12] we have discussed the paradox of Einstein, Podolsky, and Rosen (EPR) [15], and we have shown that the Wu-Shaknov experiment [10]... provides an experimental confirmation of the features of quantum mechanisms which are the basis of the above paradox" [14].

It should be pointed out that current expositions of this subject begin with the EPR paper [15], followed with the 1957 paper of Bohm and Aharonov [12] that refers to the polarisation of two quanta, propagating in opposite directions, produced in an annihilation process (this paper [12] also includes the probability amplitude for the polarisation of entangled photons, in the form of equation (1)). Next, the famous paper of Bell [16] is mentioned followed by the literature of polarisation correlation measurements of photons.

In contemporaneous EPR research, relevant to optical experiments, the entanglement probabilities are derived from the polarisation probability amplitude given in equation (1). Then, it is shown that these probabilities do violate the relevant Bell-type inequalities [8].

This brings us to a possible explanation of why the excellent contribution to physics of Pryce and Ward [4, 5] has remained shrouded in obscurity for such a long time. First, and obviously, Pryce and Ward neglected to place their work in the context of EPR. Secondly, Ward neglected to publish in the open literature the explicit *bra-ket* quantum polarisation expressions included in his 1949 thesis and that he applied to derive his 1947 final result published in *Nature* with Pryce. In retrospect, this is not surprising given the reluctance of Ward to publish [17].

Beyond the EPR implications, and Bohm and Aharanov's interpretation, the experiment seeded from Dirac's ideas [3], discussed by Wheeler [2], illustrated and described with the correct quantum physics by Pryce and Ward [5] had all the ingredients of quantum entanglement as previously mentioned by Dalitz and Duarte [18]. On a broader perspective, the evidence presented here tends to indicate that the physics of quantum entanglement, as initiated by Dirac, discussed by Wheeler, and resolved by Pryce and Ward, and Snyder *et al.*, would still be here even in the apparent absence of interpretational questions.

Thus, the probability amplitude of entangled polarisations should be added to the other momentous J. C. Ward contributions to statistical mechanics, the Standard Model, and renormalisation theory. In this regard, the sentence: *He has drawn attention to fundamental truths, and has laid down basics principles, which physicists have followed... often without knowing it, and generally without quoting him* [19] is not hyperbole.

Note added in proof: it should be noted that the probability amplitude for entangled polarisations can also be derived, in addition to Ward's approach, using Dirac's notation, from a Hamiltonian approach and from an interferometric approach [20].

Figure 2. R. H. "Dick" Dalitz (1925-2006) the creator of the Dalitz plots, a tool of crucial importance to high-energy physics in the search of new particles, such as the Higgs boson. Born in Dimboola, Victoria, he was educated at the Universities of Melbourne and Cambridge. Late in 1947, while at Cambridge, Dalitz independently derived the probability amplitude of entangled polarisations for counter-propagating photons and the corresponding polarisation correlations and disclosed the results at a physics seminar [9]. Ward and Dalitz met sometime in the late 1940s and remained close friends for life. Dalitz spend most of his career at Oxford.

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